

This conference is supported by the Syracuse University Department of Mathematics and the Graduate Student Organization.

We would like to express our thanks to all the people who helped us in making this conference possible, especially the Department of Mathematics here at Syracuse University. We would like to extend special thanks to our departmental office staff, without whom this conference would not be possible.

We would also like to thank the graduate students involved in putting this conference together. The main organizers from the MGO Board are: Rachel Gettinger, Jennifer Edmond, Casey Necheles, Stephen Farnham, and Caleb McWhorter.

Lastly, we would like to thank you all for coming and we hope that your stay at Syracuse University is as pleasant as possible.

Time	CARNEGIE 122	CARNEGIE 120
8:00 - 9:20	Breakfast and Registration (Carnegie 111)	
9:20 - 9:30	Welcome: Uday Banerjee (Carnegie 122)	
9:30 - 10:30	<p style="text-align: center;"><b>Robert Ghrist</b>  <i>Sheaves and Topological Inference from Data</i></p> <p style="text-align: center;">Carnegie 122</p>	
10:40 - 11:10	Josh Stangle <i>Decision Making with Mathematics: Prison Rules</i>	Steve Scheirer <i>Topological Complexity of Tree Configuration Spaces</i>
11:15 - 11:45	Casey Necheles <i>A Brief Introduction to Khovanov Homology</i>	James Heffers <i>Algebraic Geometric Properties of Upper Level Sets of Lelong Numbers of Closed Positive Bidimension <math>(p, p)</math> Currents on <math>n</math>-dimensional Complex Projective Space... and other fun math related things...</i>
11:50 - 12:20	Rebecca RG <i>Using Closure Operations to Study Singularities</i>	Xiaoxia Liu <i>Implicit Fixed-Point Proximity Algorithm and its Applications</i>
12:20 - 1:20	Lunch (Carnegie 111)	
1:20 - 2:20	<p style="text-align: center;"><b>Robert Connelly</b>  <i>Flexible Surfaces and Syracuse University</i></p> <p style="text-align: center;">Carnegie 122</p>	
2:30 - 3:00	Eric Ottman <i>Axiomatic Number Theory and Gödel's Incompleteness Theorems</i>	Robert Short <i>The Relative Topological Complexity of a Pair</i>
3:05 - 3:35	Bryan Goldberg <i>Joint Spectrum of the Dihedral Group</i>	Erin Griffin <i>The Isometric Immersion of a Plane Into Hyperbolic Space</i>
3:40 - 3:50	Coffee Break (Carnegie 111)	
3:55 - 4:25	Laura Ballard <i>An Introduction to Determinantal Rings</i>	Stephen Farnham <i>Estimating Spectral Densities via Principle Component Analysis</i>
4:30 - 5:00	Caleb McWhorter <i>Adic and Perfectoid Spaces</i>	Erin Tripp <i>A Faster Algorithm for Low Rank Matrix Recovery</i>

**Opening Address: Sheaves and Topological Inference from Data**

*Robert Ghrist, University of Pennsylvania*

*9:30am, Saturday, April 8*

Abstract: In this talk, I'll argue that the recent advances in applied algebraic topology and topological data analysis point to sheaf theory as a good source of mathematical structure for modeling data tethered to spaces; and cohomology as an especially useful compression of such data. I'll survey a few cutting-edge applications ranging from neuroscience to pursuit-evasion and tracking, closing with how computational issues arise and are addressed with novel mathematical perspectives.

**Keynote Address: Flexible Surfaces and Syracuse University**

*Robert Connelly, Cornell University*

*1:20pm, Saturday, April 8*

Abstract: Many years ago I was a visitor at Syracuse, and while I was there I decided to continue to work on a conjecture that said that a triangulated sphere was rigid in 3-space. At the end of my stay I found a counterexample. I will show how to construct this example and bring models.

**Decision Making with Mathematics: Prison Rules**

*Josh Stangle, Syracuse University, 10:40-11:10, Rm 122*

Abstract: This talk will be about using mathematics to solve problems vs. using mathematics to make decisions. A classic riddle will be presented and solutions will be discussed. We will examine two "optimal" solutions and the expected value of the time required to implement them, and discuss a "non-optimal" strategy and why it may make more sense to implement this. The talk should be very accessible.

**Topological Complexity of Tree Configuration Spaces**

*Steve Scheirer, Lehigh University, 10:40-11:10, Rm 120*

Abstract: The topological complexity of a path-connected space  $X$ , denoted  $TC(X)$ , is an integer which can be thought of as the minimum number of continuous "rules" required to describe how to move between any two points of  $X$ . We will consider the case in which  $X$  is a space of configurations of  $n$  points on a tree  $\Gamma$ . There are two such configuration spaces: in the first, denoted  $C^n(\Gamma)$ , the order of the points on  $\Gamma$  is of importance, while in the second, denoted  $UC^n(\Gamma)$ , the order of the points is irrelevant. We will discuss methods to determine the topological complexity of these spaces for any tree  $\Gamma$  and many values of  $n$ .

**A Brief Introduction to Khovanov Homology**

*Casey Necheles, Syracuse University, 11:15-11:45, Rm 122*

This talk will provide an introduction to Khovanov Homology and its relationship to the Jones Polynomial, complete with examples. Material will mostly be drawn from Dror Bar-Natan's 2002 paper, *On Khovanov's categorification of the Jones polynomial*. This talk should be accessible to all graduate students.

**Algebraic Geometric Properties of Upper Level Sets of Lelong Numbers of Closed Positive Bidimension  $(p, p)$  Currents on  $n$ -dimensional Complex Projective Space... and other fun math related things...**

*James Heffers, Syracuse University, 11:15-11:45, Rm 120*

Abstract: Lelong numbers are a useful tool for complex analysts who want to look at the mass a current  $T$  has at a given point. In this talk we look at the geometric properties of sets of points where a current  $T$  has “large” Lelong numbers, and see that the points where our current has large Lelong number can be contained in a small subspace of  $\mathbb{P}^n$ . The talk will start with introductory definitions and some simple examples to give the audience an intuition for these concepts before building up to the main results. The only prerequisite knowledge necessary for this talk is the ability to count!

**Using Closure Operations to Study Singularities**

*Rebecca RG, Syracuse University, 11:50-12:20, Rm 122*

In characteristic  $p > 0$ , many of the existing results on the singularities of commutative rings were proved using tight closure, a technique developed by Mel Hochster and Craig Huneke. There are also a number of results in equal characteristic 0 that have used reduction to characteristic  $p$  to take advantage of tight closure methods. In this talk, I will discuss a generalization of tight closure called a Dietz closure. The simplest Dietz closures come from tensor products with big Cohen-Macaulay modules and algebras. I will present results linking Dietz closures to singularities of commutative rings in various characteristics, and describe their relevance to the homological conjectures.

**Implicit Fixed-point Proximity Algorithms and its Applications**

*Xiaoxia Liu, Syracuse University, 11:50-12:20, Rm 120*

Abstract: In this talk, we will discuss the fixed-point proximity framework for solving convex non-smooth optimization problems with one linear composite term. Under this framework, we further propose an implicit algorithm, which involves solving implicit equations via inner contraction mappings. The performance of the proposed implicit algorithm will be demonstrated by its applications on image denoising.

### **Axiomatic Number Theory and Gödel's Incompleteness Theorems**

*Eric Ottman, Syracuse University, 2:30-3:00, Rm 122*

Abstract: At the start of the 20th century, various mathematicians attempted to remove all paradoxes from mathematics by creating systems of “axiomatic” set theory and number theory, consisting (vaguely speaking) of a set of axioms and a strict set of rules on how these axioms may be manipulated to produce theorems. However, in 1931, Kurt Gödel proved that any sufficiently powerful such axiomatic system must be either incomplete or inconsistent - in other words, in any such system there must exist either a provable false statement or an unprovable true statement. In this talk, I will attempt to give a more precise description of what I mean by “axiomatic systems,” and then to summarize the (constructive) proof of Gödel's result.

### **The Relative Topological Complexity of a Pair**

*Robert Short, Lehigh University, 2:30-3:00, Rm 120*

Abstract: In the early 2000s, Michael Farber defined the homotopy invariant “topological complexity”, denoted  $TC(X)$ , for any topological space  $X$ . He connected this to motion planning problems in robotics and named the resulting field Topological Robotics. For a subspace  $A \subset X$ , we define the relative topological complexity of the pair  $(X,A)$ , denoted  $TC(X,A)$ , to be the minimal number of motion planning rules required to move from points in  $X$  to points in  $A$ . This is a special case of a more general notion introduced by Farber. We study  $TC(X,A)$  for various interesting pairs  $(X,A)$ .

### **Joint Spectrum of the Dihedral Group**

*Bryan Goldberg, University at Albany, 3:05-3:35, Rm 122*

Abstract: For a tuple  $A = (A_1, A_2, \dots, A_n)$  of elements in a unital Banach Algebra algebra  $\mathcal{B}$  its *projective joint spectrum*  $P(A)$  is the collection of  $z \in \mathbb{C}^n$  such that  $A(z) = z_1 A_1 + z_2 A_2 + \dots + z_n A_n$  is not invertible. Using the fundamental form  $\Omega_A = -\omega_A^* \wedge \omega_A$  where  $\omega_A(z) = A^{-1}(z) dA(z)$ , Douglas and Yang defined a Hermitian metric on  $P^c(A)$ . In this talk we consider the infinite dihedral group  $D_\infty = \langle a, t : a^2 = t^2 = 1 \rangle$  with respect to the left regular representation  $\lambda$ . We'll let  $P(R) = \{z \in \mathbb{C}^2 : 1 + z_1 \lambda(a) + z_2 \lambda(t) \text{ not invertible}\}$  then we'll show that the completion of  $P^c(R)$  with respect to the Hermitian metric is  $\mathbb{C}^2 \setminus \{(\pm 1, 0), (0, \pm 1)\}$ .

**The Isometric Immersion of a Plane Into Hyperbolic Space**

*Erin Griffin, Syracuse University, 3:05-3:35, Rm 120*

Abstract: This paper will look at the isometric immersion of Euclidean planes into hyperbolic space. The result of said immersion is a horosphere or a hyperbolic cylinder. We will begin by looking at the Fundamental Theorem of Surfaces after understanding the second fundamental form, Weingarten's, Gauss', and Codazzi-Mainardi equations. Then, we will develop an understanding of immersion. We will then be able to examine Volkov and Vladamirova's proof of the results of isometric immersion of a plane into hyperbolic space.

**An Introduction to Determinantal Rings**

*Laura Ballard, Syracuse University, 3:55-4:25, Rm 122*

Abstract: Determinantal rings are really nice rings that can be handy to have up your sleeve for testing conjectures. In this talk, we will define what determinantal rings are and discuss some of their nice properties. We will then use some sleight of hand to compute a system of parameters for a determinantal ring.

**Estimating Spectral Densities via Principle Component Analysis**

*Stephen Farnham, Syracuse University, 3:55-4:25, Rm 120*

Abstract: In this talk, the principle component projection matrix will be discussed along with its application in determining rank, spectral densities, and eigenvalue gaps. A new algorithm will be introduced to approximate the principle component projection matrix using less computation time than the SVD method.

**Adic and Perfectoid Spaces**

*Caleb McWhorter, Syracuse University, 4:30-5:00, Rm 122*

Abstract: In 2011 while working on his thesis involving the Weight Monodromy Conjecture, Peter Scholze introduced the notion of perfectoid spaces. These ‘algebro-geometric’ objects in  $p$ -adic geometry greatly improved upon previous work of Gabber-Ramero in Almost Ring Theory, Falting’s Almost Purity Theorem, Fontaine-Wintenberger’s work in Galois groups, and of course Deligne’s Weight-Monodromy Conjecture. The foundation of these so called ‘perfectoid’ spaces are built upon adic spaces, which are built by gluing affinoid adic spaces. The purpose of this talk is to give some background on the theory of adic spaces leading to the definition of a perfectoid space. If time permits some discussion of recent results in Perfectoid Spaces, such as in the Direct Summand Conjecture or in Shimura varieties, will be addressed.

**A Faster Algorithm for Low Rank Matrix Recovery**

*Erin Tripp, Syracuse University, 4:30-5:00, Rm 120*

Abstract: Last year, a fast algorithm was introduced to perform low rank matrix recovery, a problem which has applications to image processing and principal component analysis. We have now modified this algorithm to make it even faster. I will discuss this modification and demonstrate its use in applications.